1. In a state of plane strain, the strain components associated with the x-y axes are

$$\epsilon_{xx} = 800 \times 10^{-6}$$
$$\epsilon_{yy} = 100 \times 10^{-6}$$
$$\gamma_{xy} = -800 \times 10^{-6}$$

Find the principal strains and the principal strain directions.

→ Given data,

$\epsilon_x = 800 \times 10^{-6}$

$\epsilon_y = 100 \times 10^{-6}$

$\gamma_{xy} = -800 \times 10^{-6}$

Principal Strains ϵ_1 & ϵ_2.

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_{1,2} = \frac{(800+100)10^{-6}}{2} \pm \sqrt{\left(\frac{(800-100)10^{-6}}{2}\right)^2 + \left(\frac{-800 \times 10^{-6}}{2}\right)^2}$$

$$\epsilon_{1,2} = 4.5 \times 10^{-4} \pm \sqrt{1.2 \times 10^{-7} + 1.6 \times 10^{-7}}.$$

$$\epsilon_{1,2} = 4.5 \times 10^{-4} \pm 5.3 \times 10^{-4}.$$

$\epsilon_1 = 9.8 \times 10^{-4}$ (Maximum Principal Strain)

$\epsilon_2 = -0.8 \times 10^{-4}$ (Minimum Principal Strain.

Orientation of Principal planes.

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}\right).$$

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{-800 \times 10^{-6}}{(800-100)10^{-6}}\right).$$

$\theta_p = 24°.$ (Orientation of Principal Strain direction)

2. The displacements for a rigid-body rotation through an angle β about an axis may be described by

$$u(x, y) = (\cos\beta - 1)x - (\sin\beta)y$$
$$v(x, y) = (\sin\beta)x + (\cos\beta - 1)y.$$

Find the strain tensor and rotation ω_z. What will happen to strain tensor and rotation in case of small β?

Step 1

Question Scenario:

The situation entails analyzing the displacement and resulting stress in a inflexible body due to a small rotation approximately an axis. This type of analysis is critical in fields which includes robotics, aerospace engineering, and any mechanical system regarding rotational motion

Given:

1. Displacements are defined by:
$\mathrm{u(x,y)} = (\cos\beta - 1)\mathrm{x} - \sin\beta\mathrm{y}$
$\mathrm{v(x,y)} = \sin\beta\mathrm{x} + (\cos\beta - 1)\mathrm{y}$
2. Small attitude approximation is applicable as (beta) is small.
3. Objective is to decide the rotation approximately the z-axis ((omega_z)) and strains in the frame.

Objective:

Determine the angular speed (omega_z) and evaluate the lines, especially whether they vanish under small rotations and compute their values.

1st step:

Approach to Solve the Question:

Employ the ideas of rigid-body dynamics, in particular focusing on rotational kinematics.

Utilize small attitude approximations and partial derivatives to decide traces.

Deriving Angular Velocity ω_z:

To find ω_z we use the rotation matrix form for a rotation about the z-axis:

$$\mathrm{R} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

For small angles, where β is close to zero, we approximate $(\cos\beta = 1)$ and $(\sin\beta = \beta)$:

$$\mathrm{R} = \begin{bmatrix} 1 & -\beta \\ \beta & 1 \end{bmatrix}$$

see the supplanting vector as the difference between revolve and initial position , we get :

$$u = R\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix}$$

$$u = \begin{bmatrix} 0 & -\beta \\ \beta & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Thus:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\beta y \\ \beta x \end{bmatrix}$$

1st step:

This calculation show that the supplanting Fields $(\mathbf{u})$ $\mathbf{and}$ $(\mathbf{v})$ directly correspond to the rigid-body rotation around the z-axis by an angle β, identify $\omega_z = \beta$ radians per unit time .

Answer

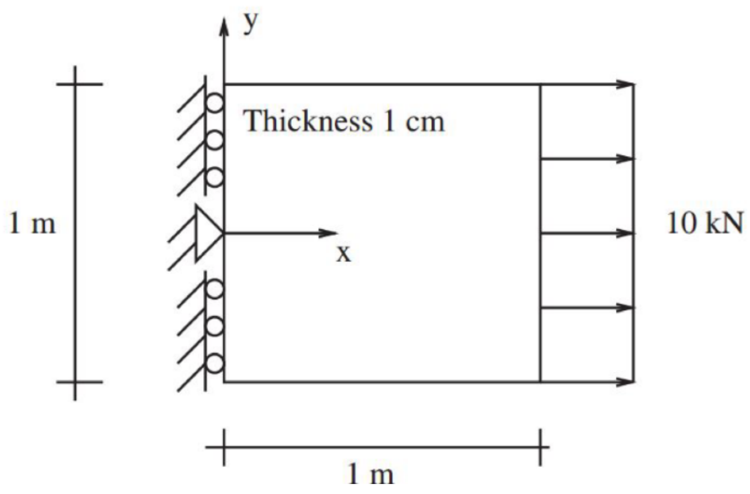
Angular Velocity (ω_z) **:** (β) radians per unit time.

Strain : All strain component ((ϵ_{xx}), (ϵ_{yy}), (γ_{XY}))be zero , bespeak no internal distortion .

Make the strain disappear ?

Yes , the strain vanish when β be small .

3. Consider a thin (1m x 1m x 1cm) steel plate which is loaded on one edge with a uniformly distributed 10 kN load and supported on the other end with a center pin and edge rollers, as shown. Assume we make a measurement of the displacement field and find that $u(x, y) = (5 \times 10^{-6})x$ m, and $v(x, y) = (0.15 \times 10^{-6})y$ m. Determine the strain field in the plate.



Step 1

calculate these strain components:

1. Longitudinal strain epsilon_{x}

$$\epsilon x = \partial u \partial x = \partial \partial x\left(510^{-6} x\right) = 5 \times 10^{-6}$$

2. Transverse strain epsilon_{y}

$$\epsilon y = \partial v \partial y = \partial \partial y\left(-0.1510^{-6} y\right) = -0.15 \times 10^{-6}$$

3. Shear strain gamma xy ;

$$\gamma xy = 12(\partial u \partial y + \partial v \partial x) = 12(0 + 0) = 0$$

Step 2

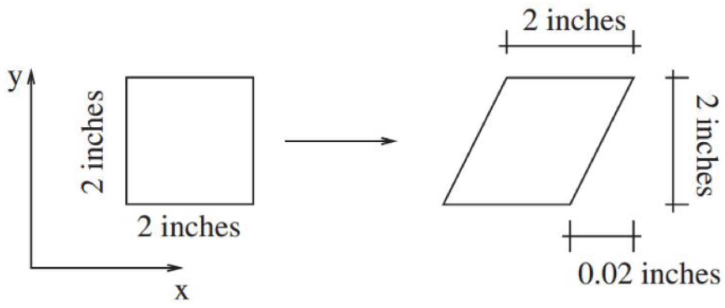
Longitudinal strain epsilon_{x} = 5 * 10 ^ - 6

- Transverse strain epsilon_{y} = - 0.15 * 10 ^ - 6
- Shear strain gamma xy =0

Answer

These strains describe how the material deforms under the given displacement field andapplied load.

4. Consider a two-dimensional body shown. The undeformed state of the body is shown on the left. After the application of load, the body takes on the configuration shown on the right. What is the average shear strain, γ_{xy}, in the body?



Step 1

Shear strain is the ratio of the displacement in the displaced layer to the dimension of the object normal to the displaced layer.
In this case, the displaced layer is the upper layer.

1st step:

As the displacement in the upper layer is small, the change in the other dimensions is negligible.

Step 2

The shear strain is calculated as follows:

$$\varepsilon = \frac{x}{L}$$

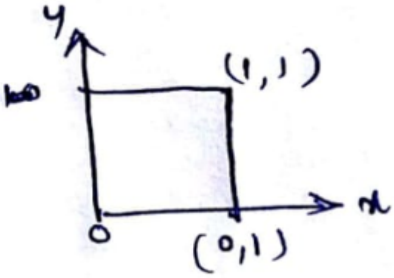
$$= \frac{0.02}{2}$$

$$= 0.01$$

5. Consider a two dimensional body occupying the region [0,1] Œ [0,1] whose displacement field is given by $u = (4x^2 + 2) \times 10^{-4}$ and $v = (2x^4 + 3y^4) \times 10^{-4}$. What is the strain field for the body? Assume the numerical constants have consistent units.

$u = (4x^2+2) \times 10^{-4}$ $\quad v = (2x^4 + 3y^4) \times 10^{-4}$

Size $[0,1] \times [0,1]$



$\varepsilon_{xx} = \frac{\partial u}{\partial x}$

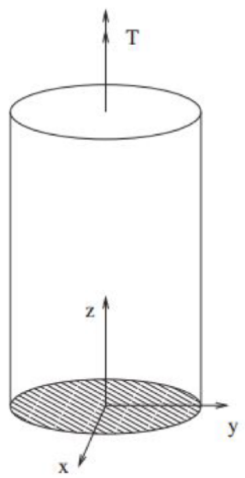
$\Rightarrow \frac{\partial}{\partial x}(4x^2+2) \times 10^{-4} = 8x \times 10^{-4}$.

at 0.2 $\quad \varepsilon_{xx} = 8 \times 0.2 \times 10^{-4} = 1.6 \times 10^{-4}$

$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(2x^4+3y^4) \times 10^{-4}$

at 0.2 $\quad = 0 + 12y^3 \times 10^{-4} \Rightarrow 9.6 \times 10^{-6}$

6. Shown below is a solid circular rod of material. The bottom is clamped, and torque is applied to the top. The motion of the material has been measured in cylindrical coordinates as $u_r = 0$, $u_\theta = \alpha r z$, $u_z = 0$, where α is a given constant with appropriate dimensions. What is the strain field in the rod? [Note: You need to use strain-displacement relation in cylindrical coordinate system]



Determine the strain field in the rod by using the relations as follows:

$$\begin{bmatrix} \varepsilon_{rr} & \frac{1}{2}\gamma_{r\theta} & \frac{1}{2}\gamma_{rz} \\ \frac{1}{2}\gamma_{r\theta} & \varepsilon_{\theta\theta} & \frac{1}{2}\gamma_{\theta z} \\ \frac{1}{2}\gamma_{rz} & \frac{1}{2}\gamma_{\theta z} & \varepsilon_{zz} \end{bmatrix}$$

Here, ε_{rr} is the relative change in length of the body in the r direction.

$$\varepsilon_{rr} = u_{r,r} = \frac{\partial u_r}{\partial r}$$

$\varepsilon_{\theta\theta}$ is the relative change in length of the body in the angular direction.

$$\varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right)$$

ε_{zz} is the relative change in length of the body in the z direction.

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$\gamma_{r\theta}$ is the change in angle between the r and θ coordinate lines at a given point.

$$\gamma_{r\theta} = \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial u_\theta / r}{\partial r}\right) = \gamma_{\theta r}$$

γ_{rz} is the change in angle between the r and z coordinate lines at a given point.

$$\gamma_{rz} = \left(\frac{\partial u_r}{\partial z} + r\frac{\partial u_z}{\partial r}\right) = \gamma_{zr}$$

$\gamma_{\theta z}$ is the change in angle between the θ and z coordinate lines at a given point.

$$\gamma_{\theta z} = \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) = \gamma_{z\theta}$$

Substitute all the values as follows:

$$\begin{bmatrix} \varepsilon_{rr} & \frac{1}{2}\gamma_{r\theta} & \frac{1}{2}\gamma_{rz} \\ \frac{1}{2}\gamma_{\theta r} & \varepsilon_{\theta\theta} & \frac{1}{2}\gamma_{\theta z} \\ \frac{1}{2}\gamma_{zr} & \frac{1}{2}\gamma_{z\theta} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial u_\theta / r}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + r\frac{\partial u_z}{\partial r}\right) \\ \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + r\frac{\partial u_\theta / r}{\partial r}\right) & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \\ \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + r\frac{\partial u_z}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial 0}{\partial r} & \frac{1}{2}\left(\frac{1}{r}\frac{\partial 0}{\partial \theta} + r\frac{\partial(\alpha r z)/r}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial 0}{\partial z} + r\frac{\partial 0}{\partial r}\right) \\ \frac{1}{2}\left(\frac{1}{r}\frac{\partial 0}{\partial \theta} + r\frac{\partial(\alpha r z)/r}{\partial r}\right) & \frac{1}{r}\left(\frac{\partial(\alpha r z)}{\partial \theta} + 0\right) & \frac{1}{2}\left(\frac{\partial(\alpha r z)}{\partial z} + \frac{1}{r}\frac{\partial 0}{\partial \theta}\right) \\ \frac{1}{2}\left(\frac{\partial 0}{\partial z} + r\frac{\partial 0}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial(\alpha r z)}{\partial z} + \frac{1}{r}\frac{\partial 0}{\partial \theta}\right) & \frac{\partial 0}{\partial z} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{1}{r}\times 0 + r\times 0\right) & \frac{1}{2}(0 + r\times 0) \\ \frac{1}{2}\left(\frac{1}{r}\times 0 + r\times 0\right) & \frac{1}{r}\left(\frac{\partial(\alpha r z)}{\partial \theta} + 0\right) & \frac{1}{2}\left(\alpha r + \frac{1}{r}\times 0\right) \\ \frac{1}{2}(0 + r\times 0) & \frac{1}{2}\left(\alpha r + \frac{1}{r}\times 0\right) & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(\alpha r) \\ 0 & \frac{1}{2}(\alpha r) & 0 \end{bmatrix}$$